Large-Amplitude Vibration of an Initially Stressed Thick Circular Plate

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In this paper, nonlinear equations of motion are derived for a transversely isotropic thick circular plate in a general state of nonuniform initial stress. The equations include the effects of transverse shear and rotary inertia. The large-amplitude flexural vibrations of simply supported and clamped thick circular plates subjected to initial stress are also investigated. The initial stress is taken to be a combination of pure bending stress plus an extensional stress in the plane of the plate. Nonlinear equations of motion are used to solve the vibration problems by the Galerkin method. In addition, the effects of various parameters on the nonlinear vibration frequencies are studied.

Nomenclature

a	= radius of circular plate
D^*	= bending rigidity of plate
E	= modulus of elasticity
F^s	= body force
G	= shear modulus
G^*	= transverse shear modulus
h	= plate thickness
K	= initial stress coefficient
$M_r, M_\theta, M_{rz}, M_{\theta z}, M_{r\theta}$	= bending moment resultant
$N_r, N_\theta, N_z, N_{r\theta}, N_{rz}, N_{\theta z}$	= stress resultant
P^s	= applied surface traction
r, θ, Z	= cylindrical coordinates
S	= transverse isotropy parameter
t	= time
T	= nondimensionalized time
$u^s, \bar{u}^s$	= initial and perturbing displacement
u, v	= in-plane displacement of r and θ direction
U	= nondimensionalized plate displacement in r direction
w	= transverse deflection
W	= nondimensionalized transverse deflection
y	= r/a
β	= ratio of bending stress to normal stress
κ^2	= shear correction factor
ν	= Poisson's ratio
ρ	= density
$\sigma^{ij}, \bar{\sigma}^{ij}$	= initial and perturbing stress
ψ_r, ψ_θ	= angular changes of lines initially normal to the neutral surface
Ω, Ω^*	= nondimensionalized linear and nonlinear frequency, respectively

Introduction

THE effects of initial stresses on vibration problems studied by past investigators have been mainly concerned with the in-plane stress of a rectangular plate.[1,2] Brunelle and Robertson studied the effects of arbitrary initial stress vibration and stability problems of a thick rectangular plate.[3,4] The effect of temperature on the large-amplitude vibrations of circular plates has been studied by using the Rayleigh-Ritz method.[5] Also, Raju and Rao[6] have used the finite element method to solve nonlinear frequencies and the critical in-plane loads of a circular plate with initial thermal stresses generated due to a specified uniform temperature rise in the plate. However, they solved the problem based on the thin-plate theory.

The authors[7] have derived the governing equations for a transversely isotropic circular thick plate in a general state of nonuniform initial stress where the effects of transverse shear and rotary inertia are included. The natural frequencies of clamped and simply supported circular plates have been obtained to compare with Irie's[8] results, which are based on the Mindlin plate theory.[9,10]

The large-amplitude vibrations of plates have been discussed by a number of investigators.[11-15] However, in all of these analyses the effects of the transverse shear deformation and rotary inertia have been neglected. Recently, Raju[16] has studied the effects of geometric nonlinearity, shear deformation, and rotary inertia on axisymmetric vibrations of circular plates. The finite element method with an appropriate linearization of the nonlinear strain-displacement relations has been used. Nonlinear flexural vibrations of clamped moderately thick circular isotropic plates have been studied by Sathyamoorthy[17] with an approximation originally due to Berger.[18] There have been few studies to investigate the effects of an arbitrary state of initial stress on large-amplitude vibration problems of thick circular plates.

In this paper, nonlinear equations of motion are derived for a transversely isotropic thick circular plate in an arbitrary state of initial stresses. The effects of transverse shear deformation and rotary inertia are included in these equations. The equations are derived by using the average stress method on the basis of von Kármán's assumptions. The nonlinear vibration problems are studied by using the Galerkin approximate method. The results are compared with the results of Raju[16] and Sathyamoorthy.[17]

Perturbed Equations

We shall consider a body in a state of nonuniform initial stress, which is in static equilbirum and is subjected to a time-varying incremental deformation. Following a technique described by Bolotin,[19] the authors derived the nonlinear equations in rectangular plates[20] and linear equations in circular plates[7] by using the perturbing technique and the average stress method. In order to present the equations of the nonlinear theory in a form suitable for any system of cur-

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vilinear coordinates, we use the contravariant index of tensor analysis.[19] The nonlinear governing equations in the same form as Eqs. (1) and (2) of Ref. 7 are obtained,

$$\partial(\sigma^{ij}\partial\bar{u}^s/\partial x^j)/\partial x^i + (\partial\bar{\sigma}^{ij}\partial\bar{u}^s/\partial x^j)/\partial x^i$$
$$+\partial\bar{\sigma}^{is}/\partial x^i + \bar{F}^s + \Delta F^s = \rho\ddot{\bar{u}}^s \tag{1}$$

$$\bar{P}^s + \Delta P^s = (\sigma^{ij}\partial\bar{u}^s/\partial x^j + \bar{\sigma}^{ij}\partial u^s/\partial x^j + \bar{\sigma}^{is})n_i \tag{2}$$

It is assumed that the initial displacement gradients are so small that the product $\bar{\sigma}^{ij}\partial u^s/\partial x^j$ can be neglected. For the plate theory of large deflection, von Kármán's assumptions are employed. Therefore, the terms $\bar{\sigma}^{ij}\partial\bar{u}^s/\partial x^j$ may be dropped except for $\bar{\sigma}^{i1}\partial\bar{u}^3/\partial x^1$ and $\bar{\sigma}^{i2}\partial\bar{u}^3/\partial x^2$. In order to give clarity to the integration procedure, it is useful to partially write out Eq. (1),

$$(\partial\sigma^{ij}\partial\bar{u}^1/\partial x^j)/\partial x^i + \partial\bar{\sigma}^{i1}/\partial x^i + \bar{F}^1 + \Delta F^1 = \rho\ddot{\bar{u}}^1 \tag{3}$$

$$(\partial\sigma^{ij}\partial\bar{u}^2/\partial x^j)/\partial x^i + \partial\bar{\sigma}^{i2}/\partial x^i + \bar{F}^2 + \Delta F^2 = \rho\ddot{\bar{u}}^2 \tag{4}$$

$$(\partial\sigma^{ij}\partial\bar{u}^3/\partial x^j)/\partial x^i + (\partial\bar{\sigma}^{ij}\partial\bar{u}^3/\partial x^j)/\partial x^i$$
$$+\partial\bar{\sigma}^{i3}/\partial x^i + \bar{F}^3 + \Delta F^3 = \rho\ddot{\bar{u}}^3 \tag{5}$$

Governing Equations

For cylindrical polar coordinates, let ξ_r, ξ_θ, and ξ_z be the physical components of displacement and ϵ_{rr}, $\epsilon_{r\theta}$,...$\bar{\sigma}_{rr}$, $\bar{\sigma}_{r\theta}$, etc., be the physical components of the strain and stress, respectively. We obtain the following results[21]:

$$x^1 = r, \quad x^2 = \theta, \quad x^3 = Z \tag{6}$$

$$ds^2 = dr^2 + r^2 d\theta^2 + dZ^2 \tag{7}$$

$$g_{11} = 1, \quad g_{22} = r^2, \quad g_{33} = 1, \quad \text{all other } g_{ij} = 0 \tag{8}$$

$$g^{11} = 1, \quad g^{22} = 1/r^2, \quad g^{33} = 1, \quad \text{all other } g^{ij} = 0 \tag{9}$$

The Euclidean Christoffel symbols are

$$\Gamma^1_{22} = -r, \quad \Gamma^2_{12} = \Gamma^2_{21} = 1/r, \quad \text{all other } \Gamma^i_{jk} = 0 \tag{10}$$

The strain-displacement relations are

$$\epsilon_{rr} = \xi_{r,r} + \xi^2_{z,r}/2, \quad \epsilon_{\theta\theta} = (\xi_r + \xi_{\theta,\theta})/r + \xi^2_{z,\theta}/2r^2$$
$$\epsilon_{r\theta} = [(\xi_{r,\theta} - \xi_\theta)/r + \xi_{\theta,r} + \xi_{z,r}\xi_{z,\theta}/r]/2$$
$$\epsilon_{rz} = (\xi_{z,r} + \xi_{r,z})/2, \quad \epsilon_{\theta z} = (\xi_{\theta,z} + \xi_{\theta,\theta}/r)/2, \quad \epsilon_{zz} = \xi_{z,z} \tag{11}$$

Physical components are

$$\bar{u}^1 = \xi_r, \quad \bar{u}^2 = \xi_\theta/r, \quad \bar{u}^3 = \xi_z, \quad \sigma^{11} = \sigma_{rr}, \quad \sigma^{22} = \sigma_{\theta\theta}/r^2,$$
$$\sigma^{33} = \sigma_{zz}, \quad \sigma^{12} = \sigma_{r\theta}/r, \quad \sigma^{23} = \sigma_{\theta z}/r, \quad \sigma^{13} = \sigma_{rz} \tag{12}$$

The stress-displacement relations are given by

$$\bar{\sigma}_{rr} = E/(1-\nu^2)[\xi_{r,r} + \xi^2_{z,r}/2 + \nu(\xi_{\theta,\theta}/r + \xi_r/r + \xi^2_{z,\theta}/2)] \tag{13}$$

$$\bar{\sigma}_{\theta\theta} = E/(1-\nu^2)[\xi_{\theta,\theta}/r + \xi_r/r + \xi^2_{z,\theta}/2 + \nu(\xi_{r,r} + \xi^2_{z,r}/2)] \tag{14}$$

$$\bar{\sigma}_{r\theta} = G(\xi_{r,\theta}/r + \xi_{\theta,r} - \xi_\theta/r + \xi_{z,r}\xi_{z,\theta}/r) \tag{15}$$

$$\bar{\sigma}_{\theta z} = \kappa^2 G^*(\xi_{\theta,z} + \xi_{z,\theta}/r) \tag{16}$$

$$\bar{\sigma}_{rz} = \kappa^2 G^*(\xi_{z,r} + \xi_{r,z}) \tag{17}$$

where G^* takes into account the effects of transverse isotropy. If $G^* = G$ the material is isotropic.

The incremental displacements are assumed to be of the form,

$$\xi_r(r,\theta,Z,t) = u(r,\theta,t) + Z\psi_r(r,\theta,t) \tag{18}$$

$$\xi_\theta(r,\theta,Z,t) = v(r,\theta,t) + Z\psi_\theta(r,\theta,t) \tag{19}$$

$$\xi_z(r,\theta,Z,t) = w(r,\theta,t) \tag{20}$$

Such a displacement field is like the one Mindlin used in his derivation of the equations for a noninitially stressed thick plate.[9] u and v are the in-plane displacements and w is the lateral deflection of the neutral surface. ψ_r and ψ_θ are the rotations due to bending.

Substituting Eqs. (18-20) into Eqs. (13-17) gives,

$$\bar{\sigma}_{rr} = E/(1-\nu^2)[u_{,r} + Z\psi_{r,r} + w^2_{,r}/2 + \nu(v_{,\theta}$$
$$+ Z\psi_{\theta,\theta} + w^2_{,\theta}/2r)/r] \tag{21}$$

$$\bar{\sigma}_{\theta\theta} = E/(1-\nu^2)[(v_{,\theta} + Z\psi_{\theta,\theta} + u + Z\psi_r + w^2_{,\theta}/2r)/r$$
$$+ \nu(u_{,r} + Z\psi_{r,r} + w^2_{,r}/2)] \tag{22}$$

$$\bar{\sigma}_{r\theta} = G[(u_{,\theta} + Z\psi_{r,\theta})/r + v_{,r} + Z\psi_{\theta,r}$$
$$- (v + Z\psi_\theta)/r + w_{,r}w_{,\theta}/r] \tag{23}$$

$$\bar{\sigma}_{\theta z} = \kappa^2 G^*(\psi_\theta + w_{,\theta}/r) \tag{24}$$

$$\bar{\sigma}_{rz} = \kappa^2 G^*(\psi_r + w_{,r}) \tag{25}$$

Prior to giving the equations of motion, the following initial stress resultants and material parameters are defined:

$$N_r = \int\sigma_{rr}dZ, \qquad N_\theta = \int\sigma_{\theta\theta}dZ,$$
$$N_{rz} = \int\sigma_{rz}dZ, \qquad N_z = \int\sigma_{zz}dZ,$$
$$M_r = \int\sigma_{rr}ZdZ, \qquad M_\theta = \int\sigma_{\theta\theta}ZdZ,$$
$$M_{rz} = \int\sigma_{rz}ZdZ, \qquad M_{\theta z} = \int\sigma_{\theta z}ZdZ,$$
$$M^*_\theta = \int\sigma_{\theta\theta}Z^2dZ, \qquad M^*_{r\theta} = \int\sigma_{r\theta}Z^2dZ,$$
$$D^* = Eh^3/12(1-\nu^2), \qquad Gh^3/12 = D^*(1-\nu)/2$$

$$N_{r\theta} = \int\sigma_{r\theta}dZ$$
$$N_{\theta z} = \int\sigma_{\theta z}dZ$$
$$M_{r\theta} = \int\sigma_{r\theta}ZdZ$$
$$M^*_r = \int\sigma_{rr}Z^2dZ$$
$$D = Eh/(1-\nu^2)$$

where all of the integrals are through the thickness of the plate from $-h/2$ to $+h/2$.

Using the technique of a covariant derivative,[21] integrating Eqs. (3-5) through the thickness of the plate, and using the stress-displacement relationship will result in r-extensional, θ-extensional, and Z-shear equations as follows. Multiplying Eqs. (3) and (4) by Z, integrating through the thickness of the plate, and using the stress-displacement relationship and tensor operation will result in the r-moment and θ-moment equations shown next.

The r-extension equation is

$$D\left[u_{,r}+\nu\left(\frac{1}{r}v_{,\theta}+\frac{u}{r}\right)\right]_{,r}+\frac{Gh}{r}\left(\frac{1}{r}u_{,\theta}+v_{,r}-\frac{1}{r}v\right)_{,\theta}+\frac{D}{r}\left[u_{,r}+\nu\frac{1}{r}(v_{,\theta}+u)\right]-\frac{D}{r}\left(\frac{1}{r}v_{,\theta}+\frac{u}{r}+\nu u_{,r}\right)+(N_r u_{,r}+M_r\psi_{r,r})_{,r}$$

$$+\frac{1}{r}(N_{r\theta}u_{,\theta}+M_{r\theta}\psi_{r,\theta}-N_{r\theta}v-M_{r\theta}\psi_\theta)_{,r}+(N_{rz}\psi_r)_{,r}+\left[\frac{1}{r}(N_{r\theta}u_{,r}+M_{r\theta}\psi_{r,r})+\frac{1}{r^2}(N_\theta u_{,\theta}+M_\theta\psi_{r,\theta}-N_\theta v-M_\theta\psi_\theta)+\frac{1}{r}(N_{\theta z}\psi_r)\right]_{,\theta}$$

$$+\frac{1}{r}(N_r u_{,r}+M_r\psi_{r,r})-\frac{1}{r}\left[N_{r\theta}v_{,r}+M_{r\theta}\psi_{\theta,r}+\frac{2}{r}(N_{r\theta}v+M_{r\theta}\psi_\theta)\right]+\frac{1}{r}N_{rz}\psi_r-\frac{1}{r^2}(N_\theta v_{,\theta}+M_\theta\psi_{\theta,\theta}+N_\theta u+M_\theta\psi_r)-\frac{N_{\theta z}}{r}\psi_\theta$$

$$+\frac{D}{2}\frac{1}{2}\left(w_{,r}^2+\frac{\nu}{r^2}w_{,\theta}^2\right)+D\left[w_{,r}w_{,rr}+\nu\left(\frac{-1}{r^3}w_{,\theta}^2+\frac{1}{r^2}w_{,\theta}w_{,r\theta}\right)\right]+\frac{Gh}{r}\left(\frac{1}{r}w_{,r}w_{,\theta}\right)_{,\theta}-\frac{1}{2r}D\left(\frac{1}{r^2}w_{,\theta}^2+\nu w_{,r}^2\right)+f_r=\rho h\ddot{u} \tag{26a}$$

where

$$f_r=\int_{-h/2}^{h/2}(\bar{F}_r+\Delta F_r)\,\mathrm{d}Z+u_{,r}(\sigma_{rz}^+-\sigma_{rz}^-)+\frac{h}{2}\psi_{r,r}(\sigma_{rz}^++\sigma_{rz}^-)+\frac{1}{r}\left[u_{,\theta}(\sigma_{\theta z}^+-\sigma_{\theta z}^-)+\frac{h}{2}\psi_{r,\theta}(\sigma_{\theta z}^++\sigma_{\theta z}^-)-v(\sigma_{\theta z}^+-\sigma_{\theta z}^-)\right.$$

$$\left.-\frac{h}{2}\psi_\theta(\sigma_{\theta z}^++\sigma_{\theta z}^-)\right]+\psi_r(\sigma_{zz}^+-\sigma_{zz}^-)+(\bar{\sigma}_{rz}^+-\bar{\sigma}_{rz}^-) \tag{26b}$$

The θ-extension equation is

$$(N_r v_{,r}+M_r\psi_{\theta,r})_{,r}+\frac{1}{r}(N_{r\theta}v_{,\theta}+M_{r\theta}\psi_{\theta,\theta}+N_{r\theta}u+M_{r\theta}\psi_r)_{,r}+(N_{rz}\psi_\theta)_{,r}+\left[\frac{1}{r}(N_{r\theta}v_{,r}+M_{r\theta}\psi_{\theta,r})+\frac{1}{r^2}(N_\theta v_{,\theta}+M_\theta\psi_{\theta,\theta}+N_\theta u\right.$$

$$\left.+M_\theta\psi_r)+\frac{1}{r}N_{\theta z}\psi_\theta\right]_{,\theta}+\frac{1}{r}(N_r v_{,r}+M_r\psi_{\theta,r})+\frac{1}{r}\left[N_{r\theta}u_{,r}+M_{r\theta}\psi_{r,r}+\frac{2}{r}(N_{r\theta}u+M_{r\theta}\psi_r)\right]+\frac{1}{r}N_{rz}\psi_\theta+\frac{1}{r^2}[N_\theta u_{,\theta}+M_\theta\psi_{r,\theta}$$

$$-2(N_\theta v+M_\theta\psi_\theta)]+\frac{1}{r}N_{\theta z}\psi_r+Gh\left(\frac{1}{r}u_{,\theta}+v_{,r}-\frac{1}{r}v\right)_{,r}+\frac{2}{r}Gh\left(\frac{1}{r}u_{,\theta}+v_{,r}-\frac{1}{r}v\right)+\frac{1}{r}D\left(\frac{1}{r}v_{,\theta}+\frac{1}{r}u+\nu u_{,r}\right)_{,\theta}$$

$$+Gh\left(\frac{-1}{r^2}w_{,r}w_{,\theta}+\frac{1}{r}w_{,rr}w_{,\theta}+\frac{1}{r}w_{,r}w_{,r\theta}\right)+\frac{2}{r^2}Gh(w_{,r}w_{,\theta})+\frac{1}{r}D\left(\frac{1}{r^2}w_{,\theta}w_{,\theta\theta}+\nu w_{,r}w_{,r\theta}\right)+f_\theta=\rho h\ddot{v} \tag{27a}$$

where

$$f_\theta=\int_{-h/2}^{h/2}(\bar{F}_\theta+\Delta F_\theta)\,\mathrm{d}Z+v_{,r}(\sigma_{rz}^+-\sigma_{rz}^-)+\frac{h}{2}\psi_{\theta,r}(\sigma_{rz}^++\sigma_{rz}^-)+\frac{1}{r}\left[v_{,\theta}(\sigma_{\theta z}^+-\sigma_{\theta z}^-)+\frac{h}{2}\psi_{\theta,\theta}(\sigma_{\theta z}^++\sigma_{\theta z}^-)+u(\sigma_{\theta z}^+-\sigma_{\theta z}^-)\right.$$

$$\left.+\frac{h}{2}\psi_r(\sigma_{\theta z}^++\sigma_{\theta z}^-)\right]+\psi_\theta(\sigma_{zz}^+-\sigma_{zz}^-)+(\bar{\sigma}_{\theta z}^+-\bar{\sigma}_{\theta z}^-) \tag{27b}$$

The shear force equation is

$$(N_r w_{,r})_{,r}+\frac{1}{r}(N_{r\theta}w_{,\theta})_{,r}+\left[\frac{1}{r}(N_{r\theta}w_{,r})+\frac{1}{r^2}(N_\theta w_{,\theta})\right]_{,\theta}+\frac{N_r}{r}w_{,r}+\kappa^2G^*h(w_{,r}+\psi_r)_{,r}+\frac{1}{r}\kappa^2G^*h\left(\psi_\theta+\frac{1}{r}w_{,\theta}\right)_{,\theta}+\frac{1}{r}\kappa^2G^*h(w_{,r}+\psi_r)$$

$$+D(\epsilon_1)\left(w_{,rr}+\frac{1}{r}w_{,r}\right)+\frac{2Gh}{r}\gamma w_{,r\theta}+\frac{Gh}{r}\gamma_{,\theta}w_{,r}+\frac{Gh}{r}\gamma_{,r}w_{,\theta}+D\epsilon_{1,r}w_{,r}+\frac{D}{r^2}(\epsilon_{2,\theta}w_{,\theta}+\epsilon_2 w_{,\theta\theta})+q=\rho h\ddot{w} \tag{28a}$$

where

$$\epsilon_1=u_{,r}+\frac{1}{2}w_{,r}^2+\nu\frac{1}{r}v_{,\theta}+\nu\frac{1}{2r^2}w_{,\theta}^2\qquad\gamma=\frac{1}{r}(u_{,\theta}-v)+v_{,r}+\frac{1}{r}w_{,r}w_{,\theta}\qquad\epsilon_2=\nu\left(u_{,r}+\frac{1}{2}w_{,r}^2\right)+\frac{1}{r}v_{,\theta}+\frac{1}{2r^2}w_{,\theta}^2$$

$$q=\int_{-h/2}^{h/2}(\bar{F}_z+\Delta F_z)\,\mathrm{d}Z+w_{,r}(\sigma_{rz}^+-\sigma_{rz}^-)+\frac{1}{r}w_{,\theta}(\sigma_{\theta z}^+-\sigma_{\theta z}^-)+(\bar{\sigma}_{zz}^+-\bar{\sigma}_{zz}^-)+w_{,r}(\bar{\sigma}_{rz}^+-\bar{\sigma}_{rz}^-)+\frac{1}{r}w_{,\theta}(\bar{\sigma}_{\theta z}^+-\bar{\sigma}_{\theta z}^-) \tag{28b}$$

The r-moment equation is

$$(M_r u+M_r^*\psi_{r,r})_{,r}+(M_{rz}\psi_r)_{,r}+\frac{1}{r}[M_{r\theta}u_{,\theta}+M_{r\theta}^*\psi_{r,\theta}-(M_{r\theta}v+M_{r\theta}^*\psi_\theta)]_{,r}+\left[\frac{1}{r}(N_{r\theta}u_{,r}+M_{r\theta}\psi_{r,r})+\frac{1}{r^2}(M_\theta u_{,\theta}+M_\theta^*\psi_{r,\theta})\right.$$

$$\left.-(M_\theta v+M_\theta^*\psi_\theta)+\frac{1}{r}M_{\theta z}\psi_r\right]_{,\theta}+\frac{1}{r}(M_r u_{,r}+M_r^*\psi_{r,r})-\frac{1}{r}\left[M_{r\theta}v_{,r}+M_{r\theta}^*\psi_{\theta,r}+\frac{2}{r}(M_{r\theta}v+M_{r\theta}^*\psi_\theta)\right]-(N_{rz}u_{,r}+M_{rz}\psi_{r,r})+\frac{M_{rz}}{r}\psi_r$$

$$-\frac{1}{r^2}(M_\theta v_{,\theta}+M_\theta^*\psi_{\theta,\theta}+M_\theta u+M_\theta^*\psi_r)-\frac{1}{r}[N_{\theta z}u_{,\theta}+M_{\theta z}\psi_{r,\theta}-(N_{\theta z}v+M_{\theta z}\psi_\theta)]-\frac{M_{\theta z}}{r}\psi_\theta-N_z\psi_r+D^*\left[\psi_{r,r}+\frac{\nu}{r}(\psi_{\theta,\theta}+\psi_r)\right]_{,r}$$

$$+\frac{1}{r}\cdot\frac{Gh^3}{12}\left(\frac{1}{r}\psi_{r,\theta}+\psi_{\theta,r}-\frac{1}{r}\psi_\theta\right)_{\theta}+\frac{1}{r}D^*\left[\psi_{r,r}+\frac{\nu}{r}(\psi_{\theta,\theta}+\psi_r)\right]-\kappa^2G^*h(w_{,r}+\psi_r)-\frac{1}{r}D^*\left(\frac{1}{r}\psi_{\theta,\theta}+\frac{1}{r}\psi_r+\nu\psi_{r,r}\right)+m_r=\frac{\rho h^3}{12}\ddot{\psi}_r$$

(29a)

where

$$m_r=\int_{-h/2}^{h/2}(\bar{F}_r+\Delta F_r)Z\mathrm{d}Z+\frac{h}{2}\left[u_{,r}(\sigma_{rz}^{+}+\sigma_{rz}^{-})+\frac{h}{2}\psi_{r,r}(\sigma_{rz}^{+}-\sigma_{rz}^{-})\right]+\frac{1}{r}\frac{h}{2}\left[u_{,\theta}(\sigma_{\theta z}^{+}+\sigma_{\theta z}^{-})+\frac{h}{2}\psi_{r,\theta}(\sigma_{\theta z}^{+}-\sigma_{\theta z}^{-})\right.$$
$$\left.-v(\sigma_{\theta z}^{+}+\sigma_{\theta z}^{-})-\frac{h}{2}\psi_{\theta}(\sigma_{\theta z}^{+}-\sigma_{\theta z}^{-})\right]+\frac{h}{2}\psi_r(\sigma_{zz}^{+}+\sigma_{zz}^{-})+\frac{h}{2}(\bar{\sigma}_{rz}^{+}+\bar{\sigma}_{rz}^{-}) \tag{29b}$$

The θ-moment equation is

$$(M_r v_{,r}+M_r^*\psi_{\theta,r})_{,r}+\frac{1}{r}(M_{r\theta}v_{,\theta}+M_{r\theta}^*\psi_{\theta,\theta}+M_{r\theta}u+M_{r\theta}^*\psi_r)_{,r}+(M_{rz}\psi_\theta)_{,r}+\frac{1}{r}(M_{r\theta}v_{,r}+M_{r\theta}^*\psi_{\theta,r})_{,\theta}$$
$$+\frac{1}{r^2}[M_\theta v_{,\theta}+M_\theta^*\psi_{\theta,\theta}+(M_\theta u+M_\theta^*\psi_r)]_{,\theta}+\frac{1}{r}(M_{\theta z}\psi_\theta)_{,\theta}+\frac{1}{r}(M_r v_{,r}+M_r^*\psi_{\theta,r})+\frac{1}{r}\left[M_{r\theta}u_{,r}+M_{r\theta}^*\psi_{r,r}\right.$$
$$\left.+\frac{2}{r}(M_{r\theta}u+M_{r\theta}^*\psi_r)\right]-(N_{rz}v_{,r}+M_{rz}\psi_{\theta,r})+\frac{1}{r}M_{rz}\psi_\theta+\frac{1}{r^2}[M_\theta u_{,\theta}+M_\theta^*\psi_{r,\theta}-2(M_\theta v+M_\theta^*\psi_\theta)]-\frac{1}{r}(N_{\theta z}v_{,\theta}$$
$$+M_{\theta z}\psi_{\theta,\theta}+N_{\theta z}u+M_{\theta z}\psi_r)+\frac{1}{r}M_{\theta z}\psi_r-N_z\psi_\theta+\frac{Gh^3}{12}\left(\frac{1}{r}\psi_{r,\theta}+\psi_{\theta,r}-\frac{1}{r}\psi_\theta\right)_{,r}+\frac{2}{r}\cdot\frac{Gh^3}{12}\left(\frac{1}{r}\psi_{r,\theta}+\psi_{\theta,r}-\frac{1}{r}\psi_\theta\right)$$
$$+\frac{1}{r}D^*\left(\frac{1}{r}\psi_{\theta,\theta}+\frac{1}{r}\psi_r+\nu\psi_{r,r}\right)_{,\theta}-\kappa^2G^*h\left(\frac{1}{r}w_{,\theta}+\psi_\theta\right)+m_\theta=\frac{\rho h^3}{12}\ddot{\psi}_\theta \tag{30a}$$

where

$$m_\theta=\int_{-h/2}^{h/2}(\bar{F}_\theta+\Delta F_\theta)Z\mathrm{d}Z+\frac{h}{2}\left[v_{,r}(\sigma_{rz}^{+}+\sigma_{rz}^{-})+\frac{h}{2}\psi_{\theta,r}(\sigma_{rz}^{+}-\sigma_{rz}^{-})\right]+\frac{1}{r}\cdot\frac{h}{2}\left[v_{,\theta}(\sigma_{\theta z}^{+}+\sigma_{\theta z}^{-})+\frac{h}{2}\psi_{\theta,\theta}(\sigma_{\theta z}^{+}-\sigma_{\theta z}^{-})\right.$$
$$\left.+u(\sigma_{\theta z}^{+}+\sigma_{\theta z}^{-})+\frac{h}{2}\psi_r(\sigma_{\theta z}^{+}-\sigma_{\theta z}^{-})\right]+\frac{h}{2}\psi_\theta(\sigma_{zz}^{+}+\sigma_{zz}^{-})+\frac{h}{2}(\bar{\sigma}_{\theta z}^{+}+\bar{\sigma}_{\theta z}^{-}) \tag{30b}$$

The five boundary traction conditions for the r- and θ-constant edges are derived by the same procedure as the governing equations, as follows.

r-constant edge:

$$\bar{F}_r+\Delta F_r=N_r u_{,r}+M_r\psi_{r,r}+\frac{1}{r}N_{r\theta}u_{,\theta}+\frac{1}{r}M_{r\theta}\psi_{r,\theta}-\frac{N_r}{r}v-\frac{M_r}{r}\psi_\theta+N_{rz}\psi_r+D\left[u_{,r}+\frac{1}{2}w_{,r}^2+\nu\left(\frac{1}{r}v_{,\theta}+\frac{1}{2r^2}w_{,\theta}^2\right)+\frac{\nu}{r}u\right]$$

$$\bar{F}_\theta+\Delta F_\theta=N_r\frac{1}{r}v_{,r}+M_r\frac{1}{r}\psi_{\theta,r}+\frac{1}{r^2}N_{r\theta}(v_{,\theta}+u)+\frac{1}{r^2}M_{r\theta}(\psi_{\theta,\theta}+\psi_r)+\frac{1}{r}N_{rz}\psi_\theta+Gh\left(\frac{1}{r}u_{,\theta}+v_{,r}-\frac{1}{r}v+\frac{1}{r}w_{,r}w_{,\theta}\right)$$

$$\bar{F}_{rz}+\Delta F_{rz}=N_r w_{,r}+\frac{1}{r}N_{r\theta}w_{,\theta}+\kappa^2G^*h(w_{,r}+\psi_r)+D\left[u_{,r}+\frac{1}{2}w_{,r}^2+\frac{\nu}{r}\left(u+v_{,\theta}+\frac{1}{2r}w_{,\theta}^2\right)\right]w_{,r}+Gh\left[\frac{1}{r}(u_{,\theta}-v)+v_{,r}+\frac{1}{r}w_{,r}w_{,\theta}\right]w_{,\theta}$$

$$\bar{M}_r+\Delta M_r=M_r u_{,r}+M_r^*\psi_{r,r}+\frac{1}{r}M_{r\theta}u_{,\theta}+\frac{1}{r}M_{r\theta}^*\psi_{r,\theta}-\frac{M_r}{r}v-\frac{M_r^*}{r}\psi_\theta+M_{rz}\psi_r+D^*\left[\psi_{r,r}+\frac{\nu}{r}(\psi_{\theta,\theta}+\psi_r)\right]$$

$$\bar{M}_\theta+\Delta M_\theta=M_r\frac{1}{r}v_{,r}+\frac{1}{r}M_r^*\psi_{\theta,r}+\frac{1}{r^2}M_{r\theta}(v_{,\theta}+u)+\frac{1}{r^2}M_{r\theta}(\psi_{\theta,\theta}+\psi_r)+\frac{1}{r}M_{rz}\psi_\theta+\frac{D^*}{2}(1-\nu)\left(\frac{1}{r}\psi_{r,\theta}+\psi_{\theta,r}-\frac{1}{r}\psi_\theta\right) \tag{31}$$

θ-constant edge:

$$\bar{F}_\theta+\Delta F_\theta=\frac{N_\theta}{r^3}v_{,\theta}+\frac{M_\theta}{r^3}\psi_{\theta,\theta}+\frac{N_{r\theta}}{r^2}v_{,r}+\frac{M_{r\theta}}{r^2}\psi_{\theta,r}+\frac{N_{\theta z}}{r^2}+D\left(\frac{1}{r}v_{,\theta}+\frac{1}{r}u+\nu u_{,r}+\frac{1}{2r^2}w_{,\theta}^2+\nu\cdot\frac{1}{2}w_{,r}^2\right)$$

$$\bar{F}_r+\Delta F_r=\frac{N_\theta}{r^2}u_{,\theta}+\frac{M_\theta}{r^2}\psi_{r,\theta}+\frac{N_{r\theta}}{r}u_{,r}+\frac{M_{r\theta}}{r}\psi_{r,r}+\frac{N_{\theta z}}{r}\psi_r+Gh\left[\frac{1}{r}(u_{,\theta}-v)+v_{,r}+\frac{1}{r}w_{,r}w_{,\theta}\right]$$

$$\bar{F}_{\theta z}+\Delta F_{\theta z}=\frac{N_\theta}{r^3}N_{r\theta}w_{,\theta}w_{,r}+\kappa^2G^*h\left(\psi_\theta+\frac{1}{r}w_{,\theta}\right)+Gh\left[\frac{1}{r}(u_{,\theta}-v)+v_{,r}+\frac{1}{r}w_{,r}w_{,\theta}\right]w_{,r}+D\left(\frac{1}{r}v_{,\theta}+\frac{1}{r}u+\nu u_{,r}+\frac{1}{2r^2}w_{,\theta}^2+\frac{\nu}{2}w_{,r}^2\right)$$

$$\bar{M}_\theta+\Delta M_\theta=\frac{M_\theta}{r^3}v_{,\theta}+\frac{M_\theta^*}{r^3}\psi_{\theta,\theta}+\frac{M_{r\theta}}{r^2}v_{,r}+\frac{M_{r\theta}^*}{r^2}\psi_{\theta,r}+\frac{M_{\theta z}^*}{r^2}\psi_\theta+D^*\left(\frac{1}{r}\psi_{\theta,\theta}+\frac{1}{r}\psi_r+\nu\psi_{r,r}\right)$$

$$\bar{M}_r+\Delta M_r=\frac{M_\theta}{r^2}u_{,\theta}+\frac{M_\theta^*}{r^2}\psi_{r,\theta}+\frac{M_{r\theta}}{r}u_{,r}+\frac{M_{r\theta}^*}{r}\psi_{r,r}+\frac{M_{\theta z}}{r}\psi_r+Gh\left(\frac{1}{r}(\psi_{r,\theta}-\psi_\theta)+\psi_{\theta,r}\right) \tag{32}$$

The alternative displacement boundary conditions are as follows.

r-constant edge:

$$u=u_{rr},\quad v=u_{r\theta},\quad w=w_{rz},\quad \psi_r=\psi_{rr},\quad \psi_\theta=\psi_{r\theta} \tag{33}$$

θ-constant edge:

$$u=u_{\theta r},\quad v=u_{\theta\theta},\quad w=w_{\theta z},\quad \psi_r=\psi_{\theta r},\quad \psi_\theta=\psi_{\theta\theta} \tag{34}$$

where the quantities on the right sides are prescribed.

Example Problem

Consider a circular plate of uniform thickness h and radius a in a state of initial stress. The state of initial stress is

$$\sigma_{rr}=\sigma_n+2Z\sigma_m/h \tag{35}$$

with all other initial stresses assumed to be zero. σ_n and σ_m are taken to be constants. It is comprised of a tension (or compression) plus bending. From Eqs. (35), the only nonzero initial stress resultants and moments are

$$N_r=h\sigma_n,\quad M_r=h^2\sigma_m/6,\quad M_r^*=h^3\sigma_n/12 \tag{36}$$

The coordinate system was chosen so that the middle plane of the plate would coincide with the r-θ plane and the origin of the coordinate system begin at the center of the plate with the positive Z axis upward.

The displacement field of axisymmetric modes of vibration is simplified by

$$\xi_r(r,Z,t)=u(r,t)+Z\psi_r(r,t),\quad \xi_\theta=0,\quad \xi_z(r,Z,t)=w(r,t) \tag{37}$$

Lateral loads and body forces are taken to be zero,

$$f_r,\quad f_\theta,\quad q,\quad m_r,\quad m_\theta=0 \tag{38}$$

The equations of motion (26-30) simplify to

$$D\left(u_{,r}+\nu\frac{u}{r}\right)_{,r}+\frac{D}{r}\left(u_{,r}+\nu\frac{1}{r}u\right)-\frac{D}{r}\left(\frac{u}{r}+\nu u_{,r}\right)$$
$$+\frac{D}{2r}(1-\nu)w_{,r}^2+Dw_{,r}w_{,rr}+(N_ru_{,r}+M_r\psi_{r,r})_{,r}$$
$$+\frac{1}{r}(N_ru_{,r}+M_r\psi_{r,r})=\rho h\ddot{u} \tag{39}$$

$$(N_r,w_{,r})_{,r}+\frac{N_r}{r}w_{,r}+\kappa^2G^*h(w_{,r}+\psi_r)_{,r}$$
$$+\frac{1}{r}\kappa^2G^*h(w_{,r}+\psi_r)+D\epsilon_1\left(w_{,rr}+\frac{1}{r}w_{,r}\right)$$
$$+D\epsilon_{1,r}w_{,r}=\rho h\ddot{w} \tag{40}$$

$$(M_ru+M_r^*\psi_{r,r})_{,r}+\frac{1}{r}(M_ru_{,r}+M_r^*\psi_{r,r})$$
$$+D^*\left(\psi_{r,r}+\frac{\nu}{r}\psi_r\right)_{,r}+\frac{1}{r}D^*\left(\psi_{r,r}+\frac{\nu}{r}\psi_r\right)$$
$$-\kappa^2G^*h(w_{,r}+\psi_r)-\frac{1}{r}D^*\left(\frac{1}{r}\psi_r+\nu\psi_{r,r}\right)$$
$$=\frac{\rho h^3}{12}\ddot{\psi}_r \tag{41}$$

where

$$\epsilon_1=u_{,r}+\tfrac{1}{2}w_{,r}^2$$

For the simply supported immovable plate the boundary conditions are,

$$\bar{M}_r+\Delta M_r=M_ru_{,r}+M_r^*\psi_{r,r}+D^*\left(\psi_{r,r}+\frac{\nu}{r}\psi_r\right)=0$$
$$w=u=0\qquad \text{at } r=a$$
$$u=\psi_r=w_{,r}=0\qquad \text{at } r=0 \tag{42}$$

and for the clamped immovable plate

$$w=u=\psi_r=0\qquad \text{at } r=a$$
$$u=\psi_r=w_{,r}=0\qquad \text{ar } r=0 \tag{43}$$

Displacements of the following form satisfy the geometric boundary conditions[14,22]

$$u(r,t)=u(A_1y+A_2y^3+A_3y^5+A_4y^7)$$
$$w(r,t)=w(1+B_1y^2+B_2y^4)$$
$$\psi_r(r,t)=\psi(C_1y+C_2y^3) \tag{44}$$

where

$y=r/a$

$A_1=(5-3\nu)/6,\quad A_2=-(3-\nu)$

$A_3=2(5-\nu)/3,\quad A_4=-(7-\nu)/6$

for the simply supported plate

$B_1=-(6+2\nu)/(5+\nu),\quad B_2=(1+\nu)/(5+\nu)$

$C_1=2(6+2\nu)/(5+\nu),\quad C_2=-4(1+\nu)/(5+\nu)$

for the clamped plate

$B_1=-2,\qquad B_2=1$

$C_1=4,\qquad C_2=-4$

Equations (45-47) are obtained by substituting the assumed displacement field of Eq. (44) into the equations of motion (39-41) and solving by Galerkin's method,

$$C_{11}U+C_{13}\psi+C_{14}W^2=C_{15}\ddot{U} \tag{45}$$

$$C_{21}W+C_{22}\psi+C_{23}UW+C_{24}W^3=C_{25}\ddot{W} \tag{46}$$

$$C_{31}U+C_{32}W+C_{33}\psi=C_{35}\ddot{\psi} \tag{47}$$

where the following coefficients and nondimensional parameters are used:

$$C_{14}=4a/h[12B_2^2D_6+8B_2B_1D_4+B_1^2D_2$$
$$+(1-\nu)/2(4\cdot B_2^2D_6+4B_1B_2D_4+B_1^2D_2)]$$

$$C_{23}=2[140B_2A_4P_9+(80B_2A_3+56A_4B_1)P_7+2B_1A_1P_1$$
$$+(36B_2A_2+30A_3B_1)P_5+(8B_2A_1+12B_1A_2)P_3]h/a$$

$$C_{24}=16(B_1^3P_3+20B_2^3P_9+24B_2^2B_1P_7+9B_2B_1^2P_5)h^2/a^2$$

$$C_{15}=(A_1D_2+A_2D_4+A_3D_6+A_4D_8)/R$$

$$C_{25}=(P_1+B_1P_3+B_2P_5)/R$$

$$C_{35}=(C_1D_{12}+C_2D_{14})/R$$

$$D_0=A_1/2+A_2/4+A_3/6+A_4/8$$

$$D_2=A_1/4+A_2/6+A_3/8+A_4/10$$

$$D_4=A_1/6+A_2/8+A_3/10+A_4/12$$

$$D_6=A_1/8+A_2/10+A_3/12+A_4/14$$

$$D_8=A_1/10+A_2/12+A_3/14+A_4/16$$

$$P_1=1/2+B_1/4+B_2/6, \quad P_5=1/6+B_1/8+B_2/10$$

$$P_3=1/4+B_1/6+B_2/8, \quad P_7=1/8+B_1/10+B_2/12$$

$$D_{10}=C_1/2+C_2/4, \quad D_{12}=C_1/4+C_2/6$$

$$D_{14}=C_1/6+C_2/8, \quad D_{16}=C_1/8+C_2/10$$

$$C_{11}=KR(49A_4D_6+25A_3D_4+9A_2D_2+A_1D_0)$$
$$+48A_4D_6+24A_3D_4+8A_2D_2$$

$$C_{13}=\beta KR(9C_2D_2+C_1D_0)/6$$

$$C_{21}=KR(16B_2P_3+4P_1B_1)+S_1(16B_2P_3+4B_1P_1)$$

$$C_{22}=S_1a(4C_2P_3+2C_1P_1)/h$$

$$C_{31}=2\beta KR(49A_4D_{16}+25A_3D_{14}+9A_2D_{12}+A_1D_{10})$$

$$C_{32}=-12S_1a(4B_2D_{14}+2B_1D_{12})/h$$

$$C_{33}=KR(9C_2D_{12}+C_1D_{10})-S_1(C_2D_{14}$$
$$+C_1D_{12})/R+8C_2D_{12}$$

$$U=u/h, \quad W=w/h, \quad S=G^*/G, \quad R=h^2/12a^2,$$

$$S_1=(1-\nu)\cdot\kappa^2\cdot S/2, \quad \beta=\sigma_m/\sigma_n,$$

$$K=N_r/DR, \quad \kappa^2=\pi^2/12, \quad T^2=\rho ht^2a^4/D^*$$

The initial in-plane compressive (tensile) stress is contained in the buckling coefficient K. If K is positive, then the stress is tensile. The initial in-plane bending stress is contained in β. When $\beta=0$ there is no initial bending stress. The equations are integrated using the fourth-order Runge-Kutta method with the time interval ΔT taken as 0.001 to obtain reasonably accurate results. In each case, the initial condition is chosen as

$$U=\psi=W=U_{,T}=\psi_{,T}=0, \quad W_{,T}=W_{\max}$$

$W_{\max}$ varies from 0.1 to 1.0 to observe the different characteristics between small- and large-amplitude vibrations. The nonlinear period for one full cycle is measured as T^* and the nonlinear frequency Ω^* is computed as $\Omega^*=1/T^*$. There are no initial stresses if $K=0$, and the in-plane inertia effects are neglected by taking $C_1=0$. Then Sathyamoorthy's governing equations[17] are obtained by coordinate transformation and neglecting both the in-plane inertia effect and the last term in Eq. (40). Nonlinear frequency, in which the effects of initial stresses and in-plane inertia are neglected, is obtained to compare with Raju's and Sathyamoorthy's results. Linear frequency Ω can be calculated by assuming $C_{14}=C_{23}=C_{24}=0$.

Table 1 Results of the Ω/Ω^* compared with other results for isotropic circular clamped (c) and simply supported (ss) plates ($W=1.0, K=0.0, \nu=0.3$)

	a/h	A[a]	B[b]	C[c]	D[d]
c	5	0.8494	0.8533	0.8388	0.8369
c	10	0.8654	0.8591	0.8530	0.8544
c	20	0.8678	0.8603	0.8550	0.8591
ss	5	0.5802	0.6634		

[a] Present results. [b] Raju's results. [c] Present results based on Sathyamoorthy's assumptions. [d] Sathyamoorthy's results.

Results and Discussions

So many parameters can be varied it is difficult to present results for all cases. From the numerous problems solved, only a few typical cases will be selected for discussion.

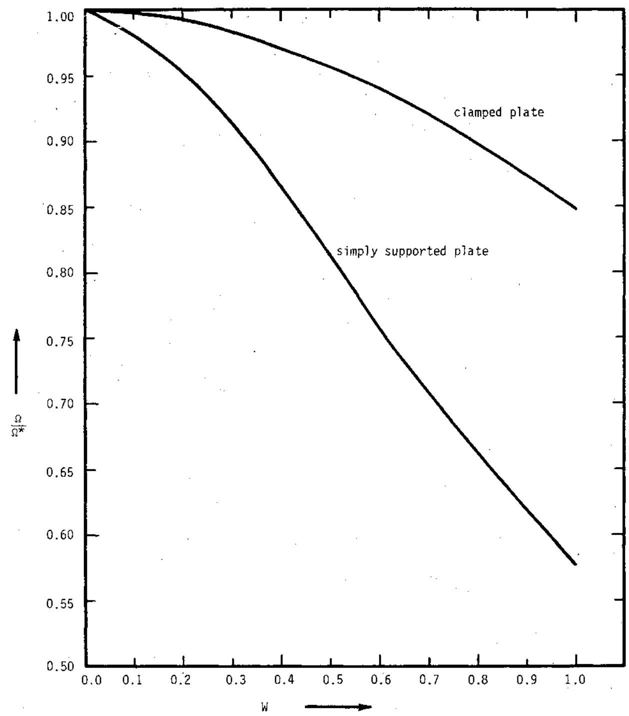

Fig. 1 Frequency ratios vs vibration amplitudes for clamped and simply supported circular plates ($a/h=5$, $K=0$, $S=1.0$, $\nu=0.3$).

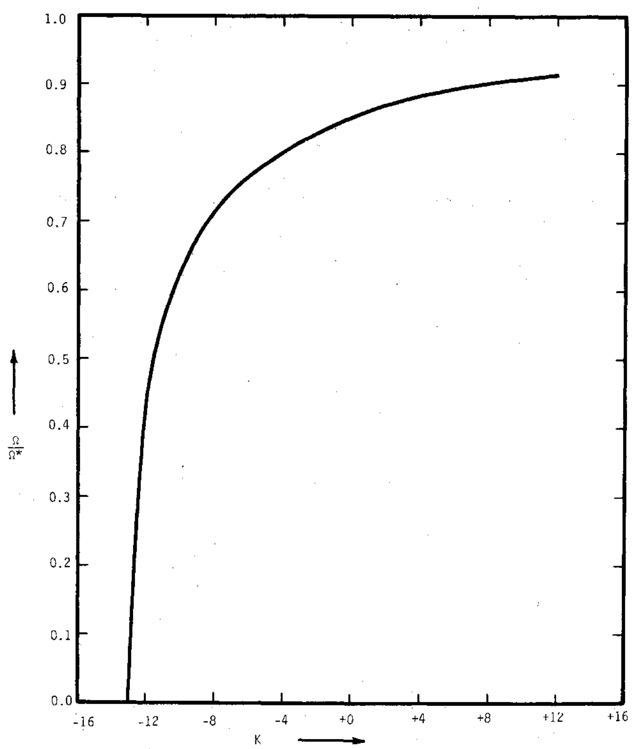

Fig. 2 Frequency ratios vs buckling coefficients for clamped circular plates ($a/h=5$, $W=1.0$, $S=1.0$, $\nu=0.3$, $\beta=0$).

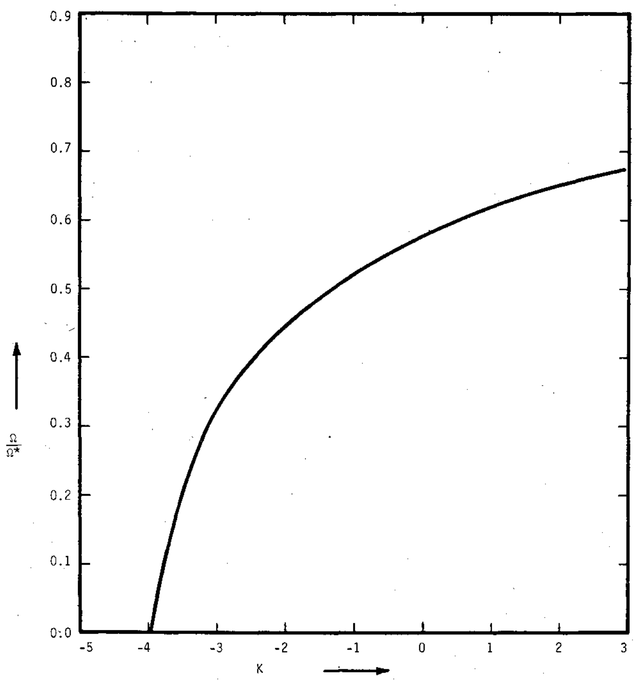

Fig. 3 Frequency ratios vs buckling coefficients for simply supported circular plates ($a/h=5$, $W=1.0$, $S=1.0$, $\nu=0.3$, $\beta=0$).

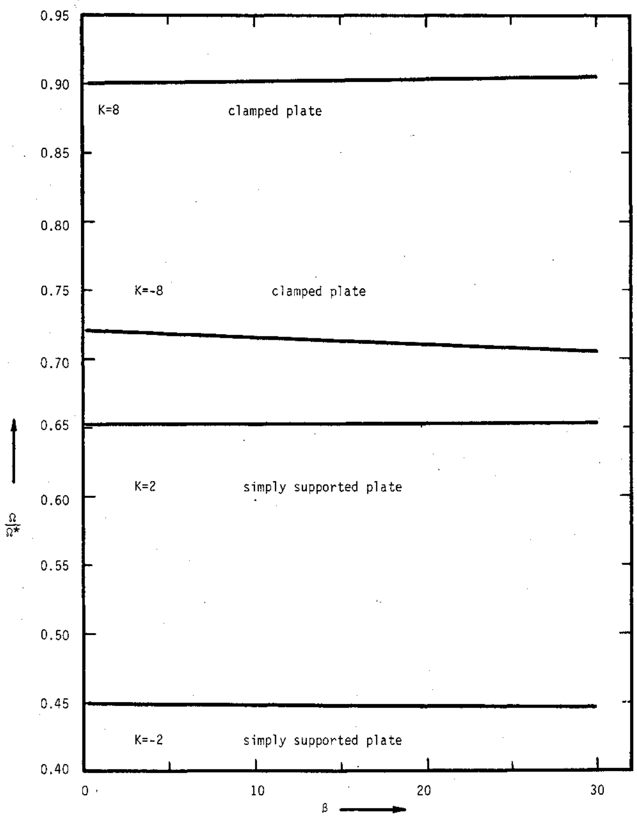

Fig. 4 Frequency ratios vs ratios of bending stress to normal stress for clampled and simply supported circular plates ($a/h=5$, $W=1.0$, $S=1.0$, $\nu=0.3$).

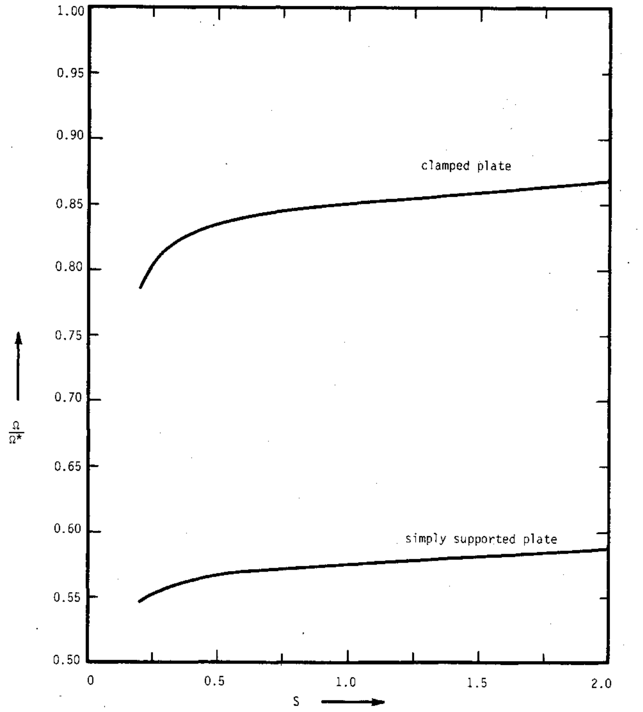

Fig. 5 Frequency ratios vs transversely isotropic parameters for clamped and simply supported circular plates ($a/h=5$, $W=1.0$, $K=0$, $\nu=0.3$, $\beta=0$).

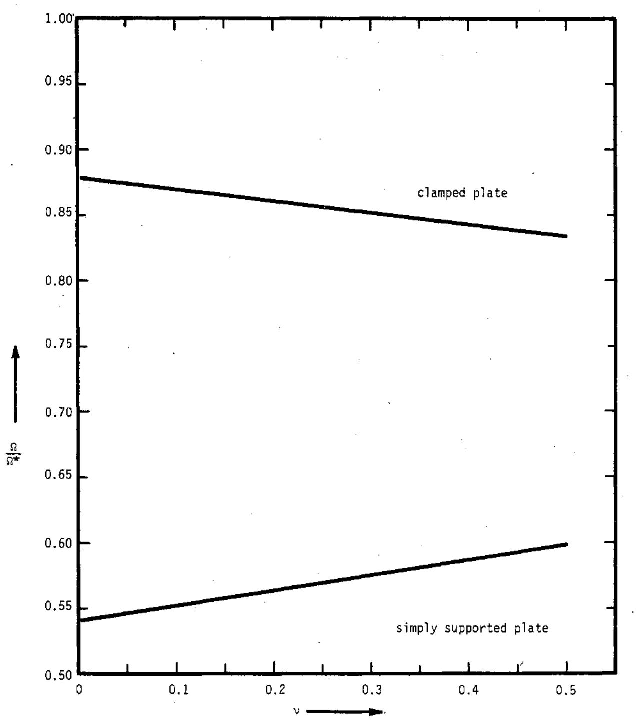

Fig. 6 Frequency ratio vs Poisson's ratio for clamped and simply supported circular plates ($a/h=5$, $W=1.0$, $S=1.0$, $K=0$, $\beta=0$).

The circular isotropic plates with no initial stresses are the first to be considered and the results are compared with those of Raju[16] and Sathyamoorthy.[17] Table 1 gives the values of the frequency ratios for various thickness ratios for the present results and others. It can be seen that the frequency ratios calculated by the present formulations coincide closely with Raju's for clamped circular plates. The effects of Sathyamoorthy's simplification are compared with the results of A and D in Table 1. It is shown that Sathyamoorthy's assumptions reduce the frequency ratios.

In the plots of Ω/Ω^* vs W in Fig. 1, a/h, K, S, and ν are chosen to be 5.0, 0.0, 1.0, and 0.3, respectively. It is observed that the frequency ratios of simply supported plates are smaller than those of clamped plates and that the differences increase as the amplitudes increase. The effects of initial stresses are studied in Figs. 2 and 3 for clamped plates and simply supported plates. It is shown that the compressive stresses produce a softening effect on frequency ratios and that the tensile loads have reverse effects. The buckling loads are obtained when the linear frequencies reduce to zero.

In the plots of Ω/Ω^* vs β in Fig. 4, a/h, W, S, and ν are equal to 5.0, 1.0, 1.0, and 0.3, respectively. The effects of the bending stresses are shown to reduce the frequency ratios when K is negative and β is positive. From the figures, it can be seen that the bending stresses have few effects on the frequency ratios. Figure 5 presents the effects of transversely isotropic coefficients S: the larger the transversely isotropic coefficient, the larger the frequency ratio. The effects of can be seen that the influence of Poisson's ratios on Ω/Ω^* have opposite effects for different boundary conditions. For a simply supported plate, Ω/Ω^* increases with increasing ν. For a clamped plate, the Poisson's ratios decrease the frequency ratios.

Conclusions

The preliminary results indicate that:

1) The nonlinear equations of motion of a thick circular plate in an arbitrary state of initial stresses have been derived in this paper.

2) Using the present governing equations one can obtain a frequency ratio of no initial stress that agrees well with Raju's and Sathyamoorthy's results for clamped plates and also obtain a lower value for simply supported plates.

3) The initial compressive stress significantly reduces the frequency ratio of large-amplitude vibrations. The tensile stresses have reverse effects.

4) The frequency ratio decreases with the increasing bending stress coefficient when K is negative.

5) The thicker the plate is, the lower the frequency ratio.

6) The frequency ratio increases with the increasing transversely isotropic coefficient.

7) For a simply supported plate, the frequency ratio increases when the Poisson's ratio increases. And the Poisson's ratio has opposite effects for a clamped circular plate.

The results presented do not cover all of the possible cases for this problem. However, they do indicate some of the many interesting effects that can be studied with the present equations. The large-amplitude vibration problems involving various boundary conditions and various edge-loading distributions of thick asymmetric circular plates are still to be investigated. Particularly interesting problems are the effects of nonconservative forces on the large-amplitude vibrations, which will be studied in the future.

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